Phase Diversity Wave Front Sensing

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Introduction

- Format for this talk:
  - Brief introduction to Adaptive Optics (AO).
  - The principle of Phase Diversity in Wavefront Sensing.
  - The current wavefront sensor and its limitations.
  - Generalised Phase Diversity (GPD) - Theory
  - Implementation.

Adaptive Optics

- Adaptive Optics (AO) refers to optical systems that can measure and compensate for optical effects introduced by the medium between the object and its image.

- AO systems are composed of 3 main parts:
  - A Deformable Mirror
  - A Wavefront Sensor
  - A Feedback loop

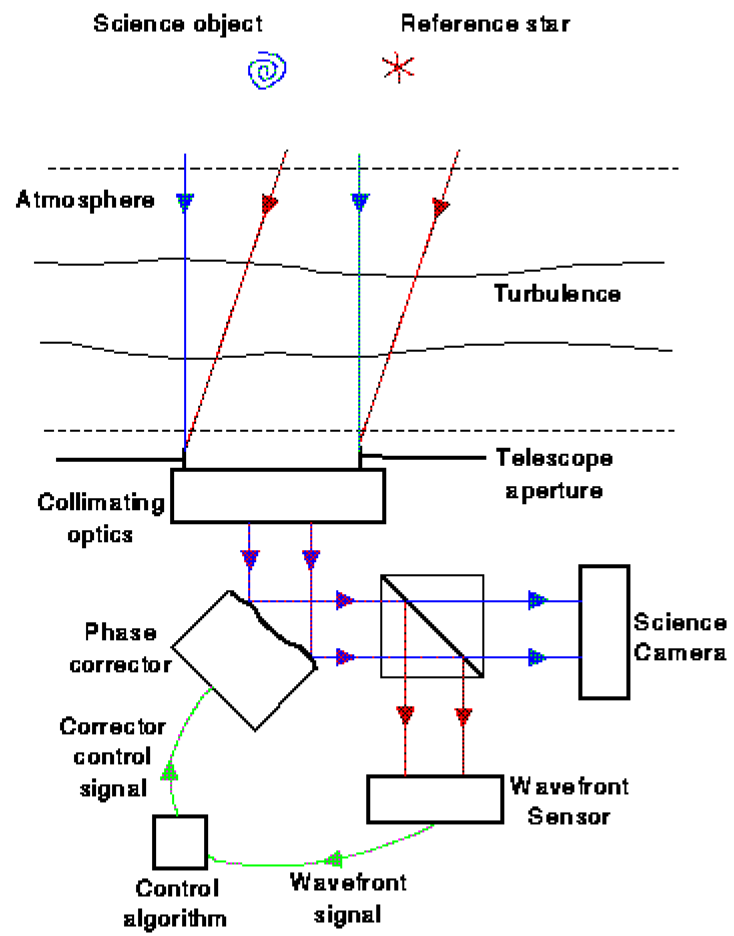


Figure 1: A telescope AO system

Zernike Polynomials

- Wavefront is a 2D map of phase.
- Wavefront mathematically described as a sum of Zernike terms.

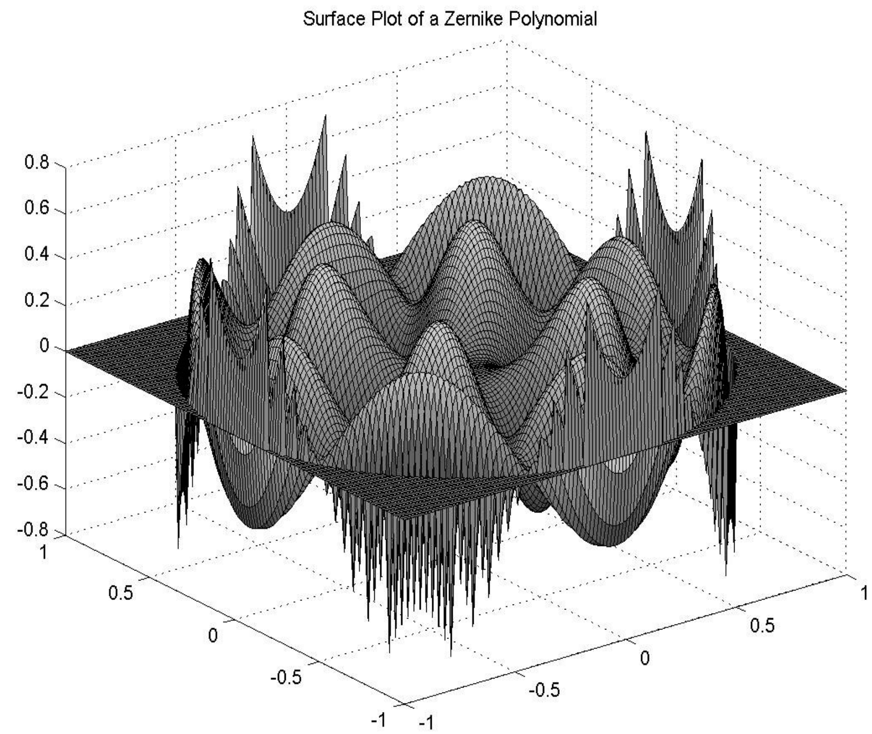


Figure 2: Surface plot of $\mathbf{Z_{10}}$

AO ~ In use today!

Images taken from the Gemini North Telescope, Mauna Kea, Hawaii

Without AO

With AO

Phase Diversity

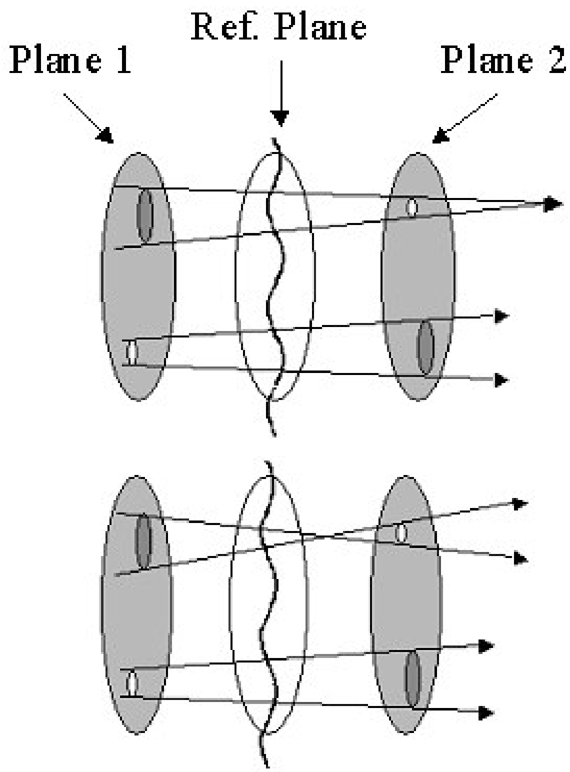


Figure 3: Two intensity planes either side of the wavefront

- DoE used to image Planes 1 & 2
- Solution of ITE gives wavefront

$$\frac{I_{\text{Plane 1}} - I_{\text{Plane 2}}}{z_1 - z_2} \sim \frac{\partial I}{\partial z}$$

$$\Psi(r) = -k \int_R dr' G(r, r') \frac{\partial I(r')}{\partial z}$$

Diffractive Optics

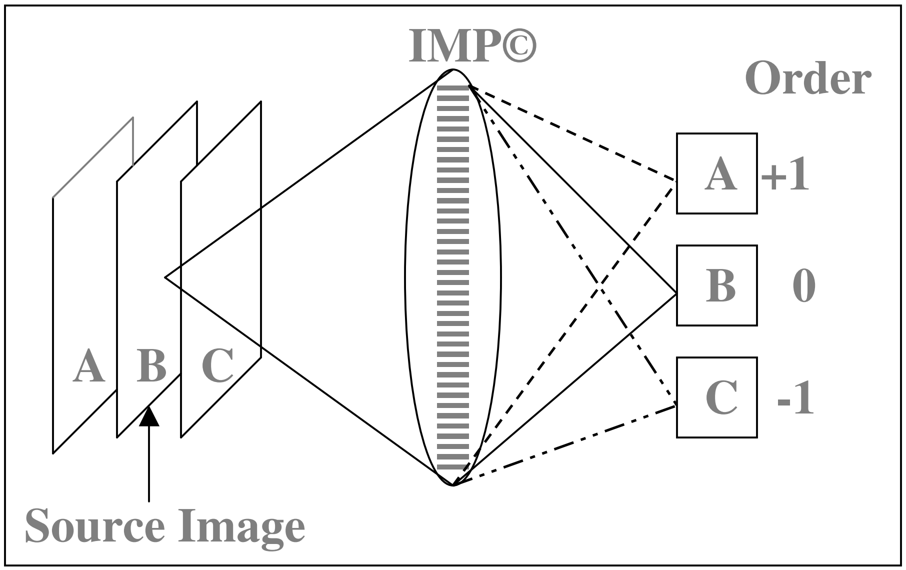
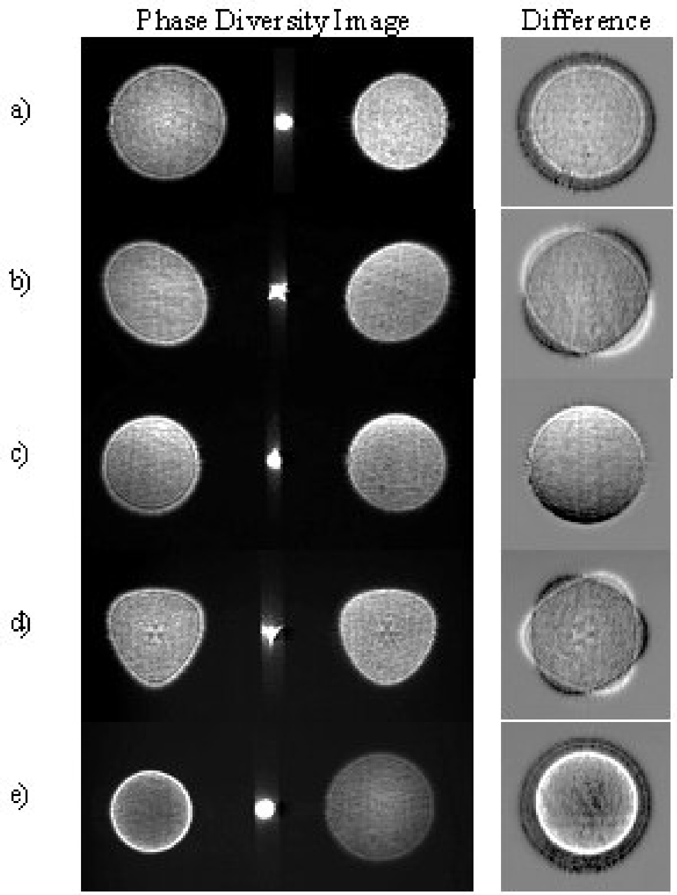


Figure 4: Shows the design of the current wavefront sensor.

Note: IMP© is a DERA (now QinetiQ) trademark

- Quadratically distorted defocus grating.
- Images of different object layers are recorded on the same focal plane.
- The plane separation and image locations are determined by the properties of the grating.

Examples of Data



- Some examples of the data seen at the focal plane.

- Easy to see the aberrations present in the data just by eye.

    - Defocus
    - Astigmatism
    - Coma
    - Trefoil
    - Spherical Aberration

Blanchard, P.M., et al., *Phase-diversity wave-front sensing with a distorted diffraction grating.* Applied Optics, 2000. **39**(35): p. 6649-6655.

Limitations

- The current Greens' function solution to the differential Intensity Transport Equation imposes several restraints on the wavefront to be reconstructed:
  - The wavefront phase must be continuous within the pupil.
  - The derivative of the wavefront phase (slope) must be continuous within the pupil.
  - The wavefront reconstruction requires computing effort and causes delay.

Generalised Phase Diversity

- GPD is required for a null sensor suitable for use with scintillated and discontinuous wavefronts.

- Images formed by convolution of the input wavefront with an aberration function (currently defocus) that has an equal but opposite aberration in the ± diffraction orders.

  – What,if anything, is unique about defocus?

  – What generic properties must an aberration function possess for use in a null sensor?

  – Can this function be optimised using *a priori* information about the wavefront to be measured?

GPD Theory - Definitions

- Complex Distribution in the entrance pupil

$$\Psi(r) = \left|\Psi(r)\right| e^{i\varphi(r)}$$

- H(ξ) and A(ξ) respectively represent the Fourier transforms of the real and imaginary parts of Ψ(r).

$$\psi(\xi) = H(\xi) + A(\xi)$$

- $F_{\pm}(\xi)$ are the filter functions:

$$\mathrm{F}_{\pm}(\xi) = \mathrm{R}(\xi) \pm i.\mathrm{I}(\xi)$$

GPD Theory - Definitions

• The detected phase-diversity intensity functions are:

$$j_{\pm}(r) = \left| \int d\xi . \psi(\xi) . \mathrm{F}_{\pm}(\xi) . e^{-i\xi r} \right|^2$$

• d(r) is the difference between the images in the ±1 diffraction order

$$\mathrm{d}(r) = 2i[\int d\xi \psi(\xi)\, \mathrm{I}(\xi) e^{-i\xi r} \int d\xi' \psi^*(\xi')\, \mathrm{R}(\xi') e^{i\xi' r}$$
$$- \int d\xi \psi(\xi)\, \mathrm{R}(\xi) e^{-i\xi r} \int d\xi' \psi^*(\xi')\, \mathrm{I}(\xi') e^{i\xi' r}]$$

Symmetries of the Filter Function

- If R and I are both odd, or both even:
- In the mixed symmetry case:

$$\frac{\mathrm{d}(r)}{2i} = \int d\xi\, \mathrm{H}(\xi)\, \mathrm{I}(\xi) e^{-ir.\xi} \int d\xi'\, \mathrm{H}^*(\xi')\, \mathrm{R}(\xi') e^{ir.\xi'} - \int d\xi\, \mathrm{H}(\xi)\, \mathrm{R}(\xi) e^{-ir.\xi} \int d\xi'\, \mathrm{H}^*(\xi')\, \mathrm{I}(\xi') e^{ir.\xi'}$$
$$+ \int d\xi\, \mathrm{H}(\xi)\, \mathrm{I}(\xi) e^{-ir.\xi} \int d\xi'\, \mathrm{A}^*(\xi')\, \mathrm{R}(\xi') e^{ir.\xi'} - \int d\xi\, \mathrm{A}(\xi)\, \mathrm{R}(\xi) e^{-ir.\xi} \int d\xi'\, \mathrm{H}^*(\xi')\, \mathrm{I}(\xi') e^{ir.\xi'}$$
$$+ \int d\xi\, \mathrm{A}(\xi)\, \mathrm{I}(\xi) e^{-ir.\xi} \int d\xi'\, \mathrm{H}^*(\xi')\, \mathrm{R}(\xi') e^{ir.\xi'} - \int d\xi\, \mathrm{H}(\xi)\, \mathrm{R}(\xi) e^{-ir.\xi} \int d\xi'\, \mathrm{A}^*(\xi')\, \mathrm{I}(\xi') e^{ir.\xi'}$$
$$+ \int d\xi\, \mathrm{A}(\xi)\, \mathrm{I}(\xi) e^{-ir.\xi} \int d\xi'\, \mathrm{A}^*(\xi')\, \mathrm{R}(\xi') e^{ir.\xi'} - \int d\xi\, \mathrm{A}(\xi)\, \mathrm{R}(\xi) e^{-ir.\xi} \int d\xi'\, \mathrm{A}^*(\xi')\, \mathrm{I}(\xi') e^{ir.\xi'}$$

$$\frac{\mathrm{d}(r)}{2i} = \int d\xi\, \mathrm{H}(\xi)\, \mathrm{I}(\xi) e^{-ir.\xi} \int d\xi'\, \mathrm{A}^*(\xi')\, \mathrm{R}(\xi') e^{ir.\xi'} - \int d\xi\, \mathrm{H}(\xi)\, \mathrm{R}(\xi) e^{-ir.\xi} \int d\xi'\, \mathrm{H}^*(\xi')\, \mathrm{I}(\xi') e^{ir.\xi'}$$
$$+ \int d\xi\, \mathrm{A}(\xi)\, \mathrm{I}(\xi) e^{-ir.\xi} \int d\xi'\, \mathrm{H}^*(\xi')\, \mathrm{R}(\xi') e^{ir.\xi'} - \int d\xi\, \mathrm{H}(\xi)\, \mathrm{R}(\xi) e^{-ir.\xi} \int d\xi'\, \mathrm{A}^*(\xi')\, \mathrm{I}(\xi') e^{ir.\xi'}$$

- Filter function must be complex valued

Necessary & Sufficient Conditions

- *Sufficient Condition*: The difference (d(*r*)) between two aberrated images is null if the input wavefront has Hermitian symmetry (I.e. is purely real) and is non-null for non-plane wavefronts.

- *Necessary Conditions:*
  - The filter function must be complex.
  - Mixed symmetries (of R and I) must not be used

Implementation

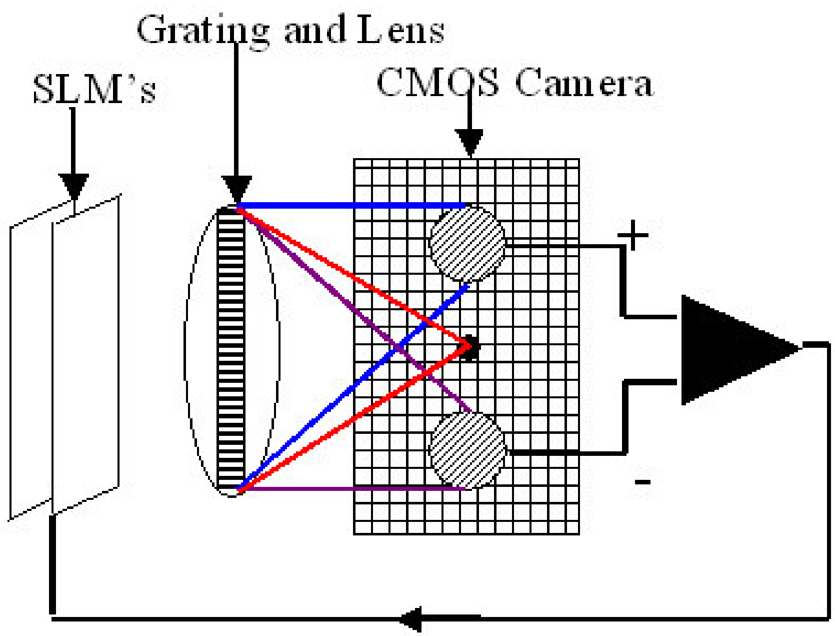


Figure 5: A suggested Compact AO System (CAOS)

- Common path aids compact design
- SLMs provide modulation.
- DoE combines phase diverse data and corrected image.
- CMOS camera

Testing

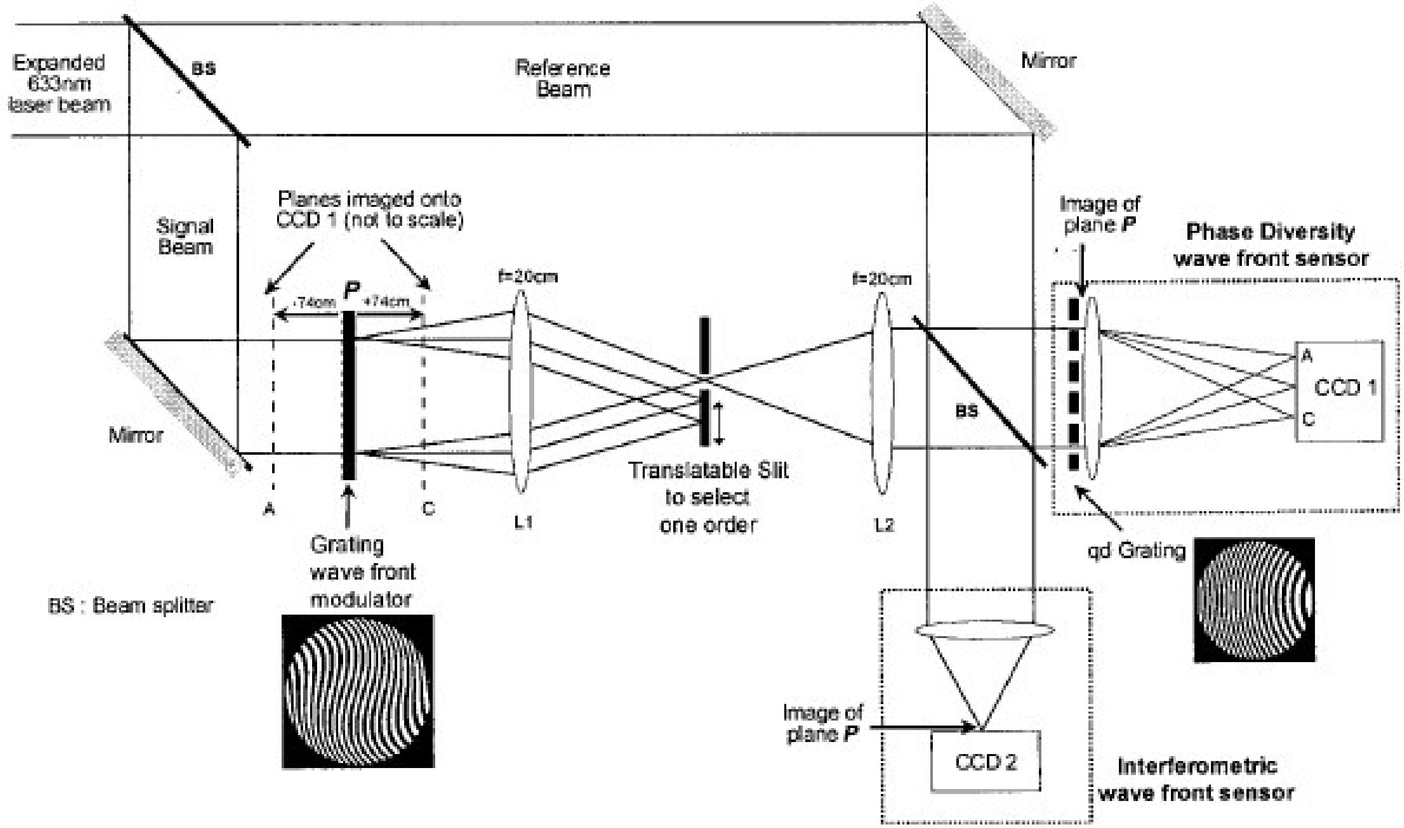


Figure 6: Mach Zender and Wavefront sensor set-up

Blanchard, P. M., D. J. Fisher, et al. (2000). "Phase-diversity wave-front sensing with a distorted diffraction grating." Applied Optics **39**(35): 6649-6655.

Future Research

- Experimental validation of the theory:
  - Manufacture of gratings for use as GPD filter functions and also to create test wavefronts with known aberrations.
  - Construction of a wavefront sensor based on these principles, using these gratings.
  - Study of optimisation when *a priori* information about the wavefront aberrations is available.
  - Implementation on a real system (WFCAM)?

Conclusions

- There is a need for a more generalised approach to PD wavefront sensing, to overcome the limitations of the current method.
- We have discovered necessary and sufficient conditions that a filter function must possess for use in a GPD based null sensor.
- Simulations that confirm this theory have been conducted.
- We have demonstrated that a compact AO system could be built based on these principles.
- Experimental testing and optimisation is to be conducted.